LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2018**

16/17UMT3MC02– VECTOR ANALYSIS AND ORDINARY DIFF. EQUATIONS

Date: 31-10-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART – A

ANSWER ALL THE QUESTIONS (10 x 2 = 20)

1. Find $\nabla\phi$ at (x, y, z) if $\phi = x^3 + y^3 + 3xyz$
2. Find the value of 'a' if the vector $(x+3y)\vec{i} + (y-2z)\vec{j} + (x+az)\vec{k}$ is solenoidal.
3. If $\vec{F} = 3xy\vec{i} - y^2\vec{j}$, evaluate $\int_C \vec{F}.d\vec{r}$ along the curve y = 2x^2 from (0,0) to (1,2).
4. Define conservative field.
5. Find a unit vector normal to the surface $x^3 + y^3 + z^3 + 3xyz = 4$ at the point (1,-2,-1).
6. State Stoke's theorem.
7. Solve $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0.$.
8. Solve $\frac{dy}{dx} + y\cos x = \frac{1}{2}\sin 2x$.
9. Solve $\left(D^2 + 5D + 6\right)y = e^{3x}$.
10. Find the particular integral of $\left(D^2 + 16\right)y = Sin4x$.

PART – B

ANSWER ANY FIVE QUESTIONS (5 x 8 = 40)

11. Prove that for any vector $\vec{F}, \nabla \times (\nabla \times \vec{F}) = \nabla(\nabla \cdot \vec{F}) - \nabla^2 \vec{F}$.
12. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |r|$, show that $\nabla.\left(r^n \vec{r}\right) = (n+3)r^n$
13. Verify Stoke's theorem for $\vec{F} = (2x-y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$, over upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy-plane.
14. Evaluate $\iiint_V \nabla.\vec{F}dv$, if $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ and V is the volume of the region bounded by x = 0, y = 0, y = 6, z = 4 and z = x^2.
15. Solve $y = xp + x\left(1 + p^2\right)^{\frac{1}{2}}$.
16. Solve $xp^2 - 2yp + x = 0$.
17. Solve $3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x.$
18. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = e^x Cos2x$

PART – C

ANSWER ANY TWO QUESTIONS (2x 20 = 40)

19. (a) Evaluate $\iint_S \vec{F}.n$ dS where $\vec{F} = z\vec{i} + x\vec{j} + 3y^2 z\vec{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between z = 0 and z = 5.

(b) Evaluate $\iint_S \vec{F}.n$ dS where $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ and S is the surface of the cube bounded by the planes x = 0, x = 1, y = 0, y = 1, z = 0, z = 1.

20. Verify divergence theorem for $\vec{A} = (x + y)\vec{\imath} + x\vec{\jmath} + z\vec{k}$ taken over region V of the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.

21. (a) Solve $\left(1-x^2\right)\frac{dy}{dx} + 2xy = x\sqrt{1-x^2}$ given that y = 0 when x = 0.

(b) Solve $y\left(xy + 2x^2y^2\right)dx + x\left(xy - x^2y^2\right)dy = 0$.

22. Solve $\frac{d^2y}{dx^2} + y = \sec x$, using variation of parameters.

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